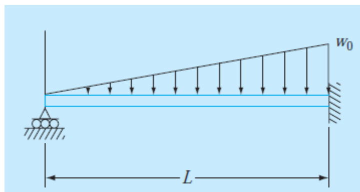
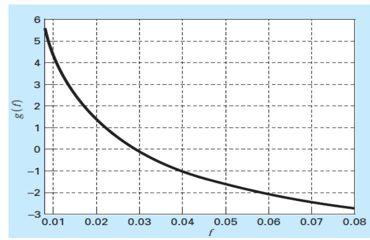
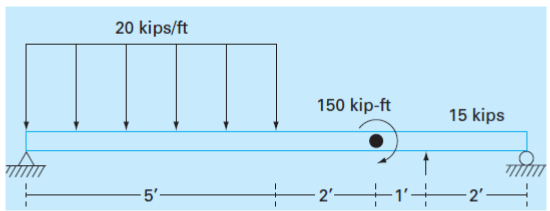
Lecture 08: Case Studies-Roots of Equations







Real World Engineering Problem

- The purpose of this lecture is to use the numerical procedures discussed in Lectures 5, 6, and 7 to solve actual engineering problems.
- Numerical techniques are important for practical applications because engineers frequently encounter problems that cannot be approached using analytical techniques.
- For example, simple mathematical models that can be solved analytically may not be applicable when real problems are involved.
- Thus, more complicated models must be employed.
- For these cases, it is appropriate to implement a numerical solution on a computer.
- In other situations, engineering design problems may require solutions for implicit variables in complicated equations.

Real World Engineering Problem

- The following case study is a representative problem you will address professionally.
- The problem is drawn from a major discipline of engineering: mechanical engineering.
- These application also serves to illustrate the trade-offs among the various numerical techniques.

Pipe Friction (Mechanical/Aerospace Engineering)

Background. Determining fluid flow through pipes and tubes has great relevance in many areas of engineering and science. In mechanical and aerospace engineering, typical applications include the flow of liquids and gases through cooling systems.

The resistance to flow in such conduits is parameterized by a dimensionless number called the *friction factor*. For turbulent flow, the *Colebrook equation* provides a means to calculate the friction factor,

$$0 = \frac{1}{\sqrt{f}} + 2.0 \log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{8.21}$$

Pipe Friction (Mechanical/Aerospace Engineering)

$$0 = \frac{1}{\sqrt{f}} + 2.0\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

where ε = the roughness (m), D = diameter (m), and Re = the *Reynolds number*,

$$\mathrm{Re} = \frac{\rho VD}{\mu}$$

where ρ = the fluid's density [kg/m^3], V = its velocity [m/s], and μ = dynamic viscosity [N · s/m^2]. In addition to appearing in Eq. (8.21), the Reynolds number also serves as the criterion for whether flow is turbulent (Re > 4000).

Pipe Friction (Mechanical/Aerospace Engineering)

$$0 = \frac{1}{\sqrt{f}} + 2.0\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

In the present case study, we will illustrate how the numerical methods covered in this part of the book can be employed to determine f for air flow through a smooth, thin tube. For this case, the parameters are $\rho = 1.23$ kg/m^3, $\mu = 1.79 \times 10^{-5}$ N · s/m^2, $D = 0.005$ m, $V = 40$ m/s and $\varepsilon = 0.0015$ mm. Note that friction factors range from about 0.008 to 0.08. In addition, an explicit formulation called the *Swamee-Jain equation* provides an approximate estimate,

$$f = \frac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

Pipe Friction (Mechanical/Aerospace Engineering)

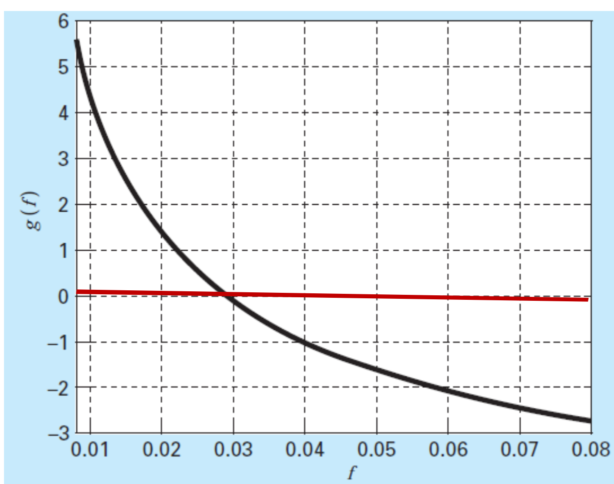
Solution. The Reynolds number can be computed as

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1.23(40)0.005}{1.79 \times 10^{-5}} = 13{,}743$$

This value along with the other parameters can be substituted into Eq. (8.21) to give

$$g(f) = \frac{1}{\sqrt{f}} + 2.0 \log\left(\frac{0.0000015}{3.7(0.005)} + \frac{2.51}{13{,}743\sqrt{f}}\right)$$

Pipe Friction (Mechanical/Aerospace Engineering)



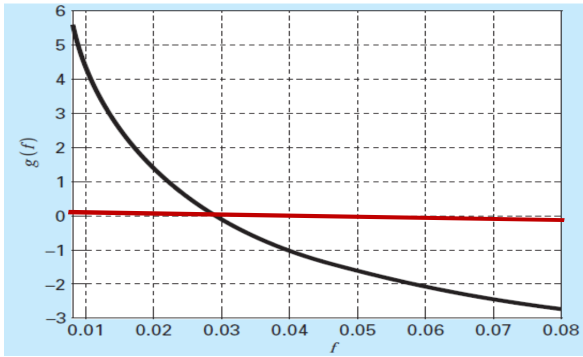
Pipe Friction (Mechanical/Aerospace Engineering)

Because we are supplied initial guesses ($x_l = 0.008$ and $x_u = 0.08$), either of the bracketing methods from Chap. 5 could be used. For example, bisection gives a value of $f = 0.0289678$ with a percent relative error of error of 5.926×10^{-5} in 22 iterations. False position yields a result of similar precision in 26 iterations. Thus, although they produce the correct result, they are somewhat inefficient. This would not be important for a single application, but could become prohibitive if many evaluations were made.

Pipe Friction (Mechanical/Aerospace Engineering)

We could try to attain improved performance by turning to an open method. Because Eq. (8.21) is relatively straightforward to differentiate, the Newton-Raphson method is a good candidate. For example, using an initial guess at the lower end of the range ($x_0 = 0.008$), Newton-Raphson converges quickly to 0.0289678 with an approximate error of 6.87×10^{-6}% in only 6 iterations. However, when the initial guess is set at the upper end of the range ($x_0 = 0.08$), the routine diverges!

Pipe Friction (Mechanical/Aerospace Engineering)



As can be seen by inspecting Fig. 8.5, this occurs because the function's slope at the initial guess causes the first iteration to jump to a negative value. Further runs demonstrate that for this case, convergence only occurs when the initial guess is below about 0.066.

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Pipe Friction (Mechanical/Aerospace Engineering)

So we can see that although the Newton-Raphson is very efficient, it requires good initial guesses. For the Colebrook equation, a good strategy might be to employ the Swamee-Jain equation (Eq. 8.22) to provide the initial guess as in

$$f = \frac{1.325}{\left[\ln\left(\frac{0.0000015}{3.7(0.005)} + \frac{5.74}{13743^{0.9}}\right)\right]^2} = 0.029031$$

For this case, Newton-Raphson converges in only 3 iterations to quickly to 0.0289678 with an approximate error of $8.51 \times 10^{-10}\%$.

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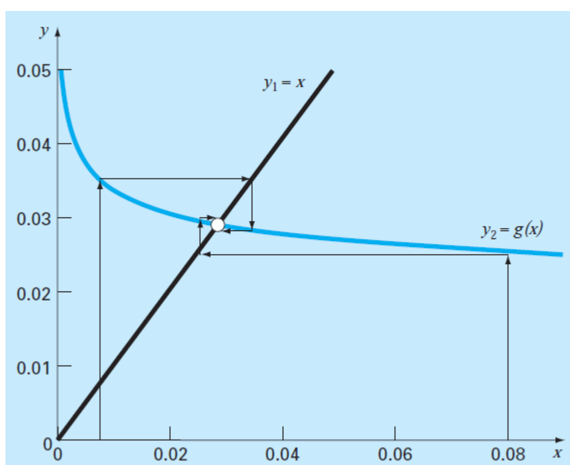
Pipe Friction (Mechanical/Aerospace Engineering)

$$0 = \frac{1}{\sqrt{f}} + 2.0 \log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

As a final note, let's see whether convergence is possible for simple fixed-point iteration. The easiest and most straightforward version involves solving for the first f in Eq. (8.21),

$$f_{i+1} = \frac{0.25}{\left(\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f_i}}\right)\right)^2} \quad (8.23)$$

Pipe Friction (Mechanical/Aerospace Engineering)



Pipe Friction (Mechanical/Aerospace Engineering)

The two-curve display of this function depicted indicates a surprising result (Fig. 8.6). Recall that fixed-point iteration converges when the y_2 curve has a relatively flat slope (i.e., $|g'(\xi)| < 1$). As indicated by Fig. 8.6, the fact that the y_2 curve is quite flat in the range from $f = 0.008$ to 0.08 means that not only does fixed-point iteration converge, but it converges fairly rapidly! In fact, for initial guesses anywhere between 0.008 and 0.08, fixed-point iteration yields predictions with percent relative errors less than 0.008% in six or fewer iterations. Thus, this simple approach that requires only one guess and no derivative estimates performs really well for this particular case.

Take-away message

- There is usually no single method that works best for all problems.
- Sophisticated users understand the strengths and weaknesses of the available numerical techniques.
- In addition, they understand enough of the underlying theory so that they can effectively deal with situations where a method breaks down.

Assignment-08

- Problems 8.10, 8.12, 8.14, 8.18, 8.33, 8.34.